Examining the
Garren-Kirk Dipole Cooling Ring with Realistic Fields

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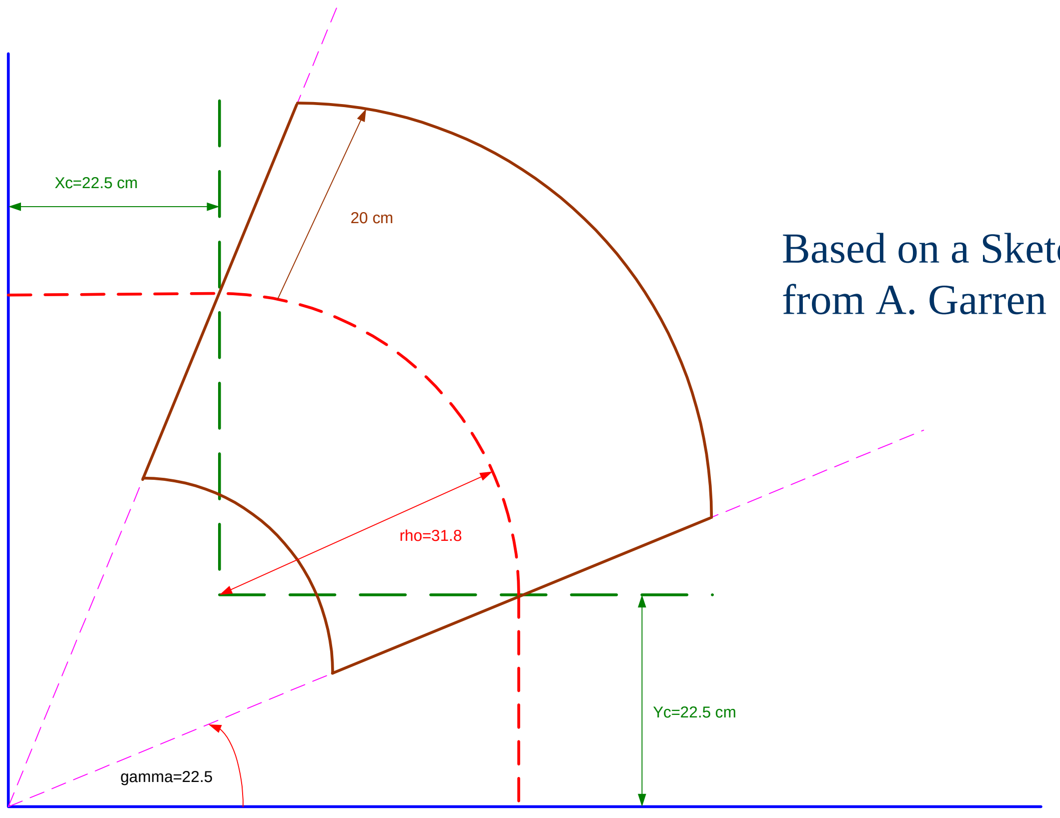
Riverside Ring Cooler

Emittance Exchange Workshop

Dipole Ring Parameters

Parameter	Value
Reference Momentum	250 MeV/c
Number of Half-Cells	4
Bend Angle per Half-Cell	90°
Ring Circumference	3.8 m
Number of RF cavities	4
RF Gradient	40 MV/m
Absorber	Pressurized H_2
Hardedge Dipole Field	2.6 T
Straight Length per Half-Cell	40 cm
Dipole Radius of Curvature	31.8 cm

Cell Geometry Description



Based on a Sketch from A. Garren

Using TOSCA

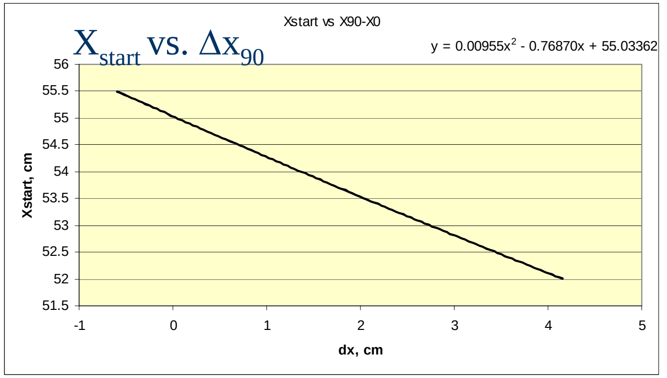
- Hard edge field calculations for the Garren-Kirk Weak Focusing Dipole Ring have shown promising results.
  - It is essential to examine the ring using realistic fields that at least obey Maxwell's equations.
- Tosca can supply fields from a coil and iron configuration.
  - We can use the program to supply a field map that can be used by ICOOL and GEANT.
- Tosca itself can also track particles through the magnetic field that it generates.
  - This allows us to avoid the descretization error that comes from field maps.

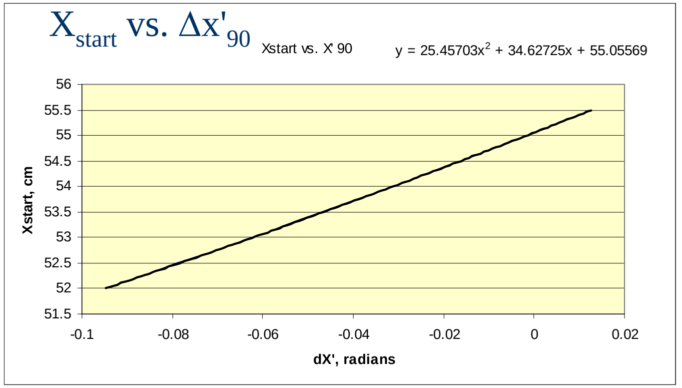
Tosca Model

- For the ease of calculation we are modeling the dipole magnets by its coils only. This may not be the way we would actually engineer the magnet if we actually built it.
  - This permits the field to be calculated with Biot-Savart integration directly. No finite-element mesh is necessary if iron is not used.
- There are limitations in the Tosca tracking.
  - Tosca permits only 5000 steps. This limits the step size to ~0.5 mm. This may limit the ultimate precision.

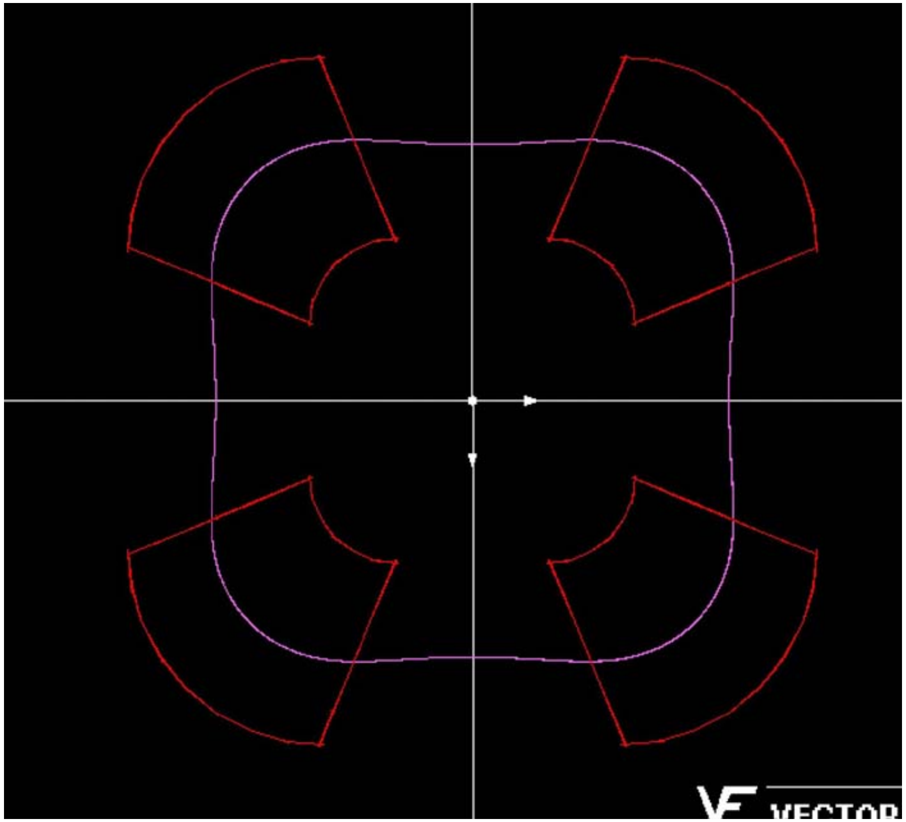
Finding the Closed Orbit

- We know that the *closed orbit* path must be in the xz *plane* and that it must have $x'=0$ at the *x-axis* from symmetry.
  - We can launch test particles with different X_{start}.
  - The figures on the right show X_{start} vs. Δx_{90} and X_{start} vs $\Delta x'_{90}$.
    - Where Δx_{90} and $\Delta x'_{90}$ are the variable differences after 90° advance.
    - We find that the best starting values are
      - X_{start}=55.03362 cm for Δx_{90}
      - X_{start}=55.05569 cm for $\Delta x'_{90}$





Closed Orbit

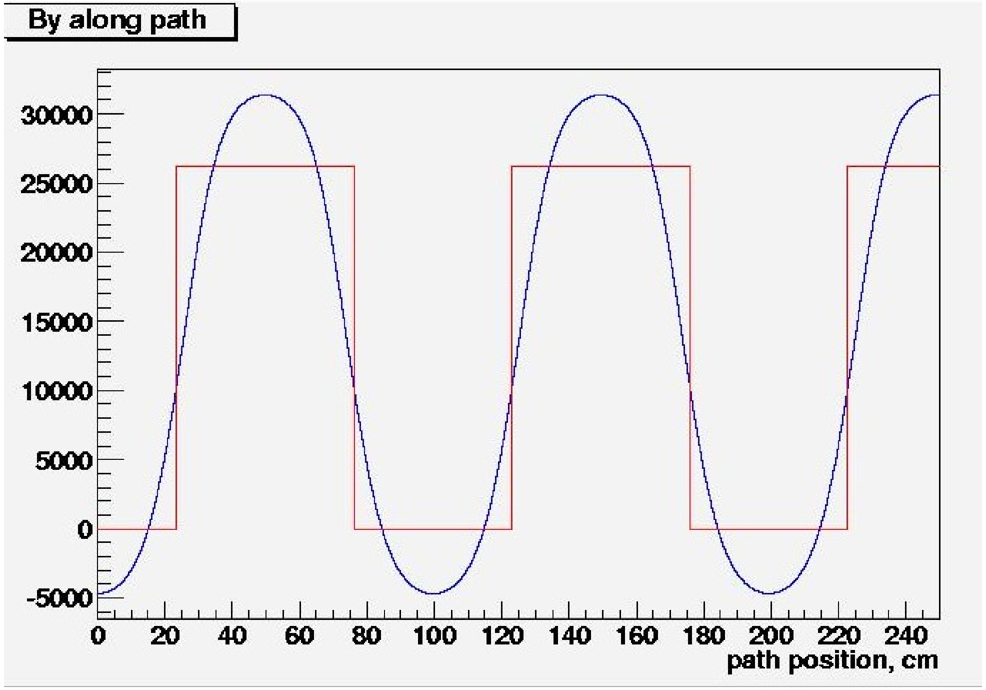


Closed orbit trajectory for 250 MeV/c μ started at x=55.02994 cm.

Note that there is curvature in region between magnets since there is still a significant field.

Field Along the Reference Path

- Figure shows B_y along the 250 MeV/c reference path.
  - The blue curve indicates the field from the Tosca field map.
  - The red curve is the hard edge field.
- Note the –0.5 T field in the gap mid-way between the magnets.



Calculating Transfer Matrices

- By launching particles on trajectories at small variations from the closed orbit in each of the transverse directions and observing the phase variables after a period we can obtain the associated *transfer matrix*.
  - Particles were launched with
    - $\delta x = \pm 1$ mm
    - $\delta x' = \pm 10$ mr
    - $\delta y = \pm 1$ mm
    - $\delta y' = \pm 10$ mr

90° Transfer Matrix

- This is the transfer matrix for transversing a quarter turn:

$$\begin{bmatrix} \delta x \\ \delta x' \\ \delta y \\ \delta y' \end{bmatrix} = \begin{bmatrix} -0.29145 & 31.965 & 0 & 0 \\ -0.0287 & -0.289 & 0 & 0 \\ 0 & 0 & -0.18336 & 52.9949 \\ 0 & 0 & -0.01823 & -0.1853 \end{bmatrix} \begin{bmatrix} \delta x_0 \\ \delta x'_0 \\ \delta y_0 \\ \delta y'_0 \end{bmatrix}$$

- This should be compared to the 2×2 matrix to obtain the twiss variables:

$$\begin{bmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ \gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{bmatrix}$$

Twiss Variables Half Way Between Magnets

Parameter	Tosca	A. Garren Synch
μ_x	98.38°	99.8784°
β_x	32.3099 cm	37.854 cm
α_x	-0.00124	0
μ_y	100.62°	92.628°
β_y	53.9188 cm	56.891 cm
α_y	0.0009894	0

Using the Field Map

- We can produce a 3D field map from TOSCA.
  - We could build a GEANT model around this field map however this has not yet been done.
  - We have decided that we can provide a field to be used by ICOOL.
    - ICOOL works in a beam coordinate system.
      - We know the trajectory of the reference path in the global coordinate system.
        - We can calculate the field and its derivatives along this path.

Representation of the Field in a Curving Coordinate System

- Chun-xi Wang has a magnetic field expansion formulism to represent the field in curved (Frenet-Serret) coordinate system.
  - This formulism is available in ICOOL.
  - Up-down symmetry kills off the a_n terms; b_s is zero since there is no solenoid component in the dipole magnets.
  - The b_n(s) are obtained by fitting

    $$B_y(x,s) = \sum b_n(s) x^n$$

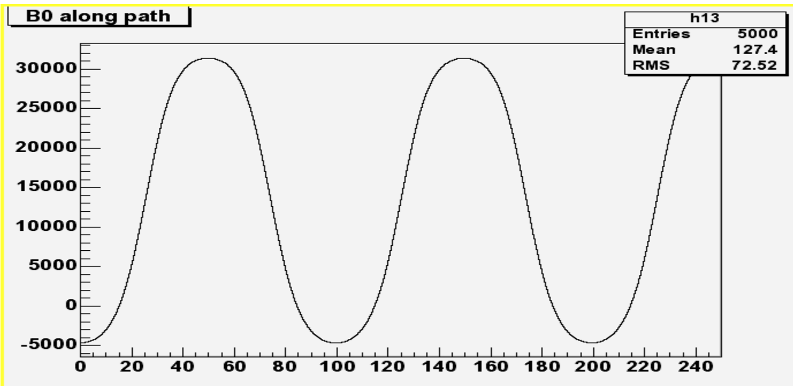
    to the field in the midplane orthogonal to the trajectory at s
    - The field is obtained from a splining the field grid.

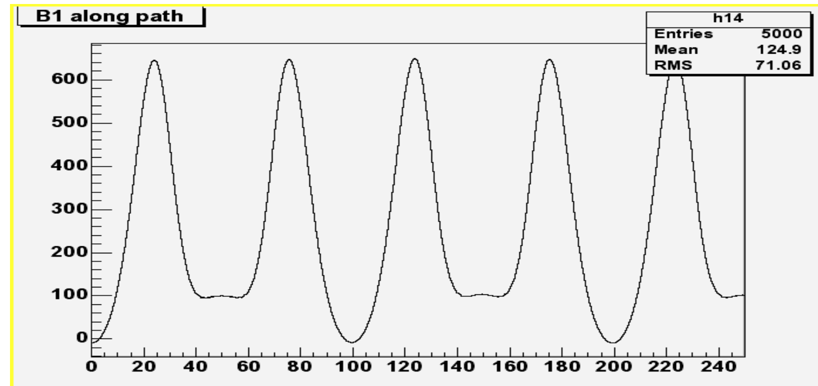
$$\begin{aligned} B_x(x,y,s) &= a_1\,x + b_1\,y + a_2\,x^2 + 2b_2\,xy \\ &-\frac{1}{2}\left[2a_2 + \kappa(a_1 - 2b_s') - \kappa' b_s\right] y^2 + a_3\,x^3 + 3b_3\,x^2 y \\ &-\frac{1}{2}\left[6a_3 + a_1'' + 2\kappa(a_2 + 3\kappa' b_s) - 2\kappa^2(a_1 - 3b_s')\right] xy^2 \\ &-\frac{1}{6}\left[6b_3 + b_1'' + 2\kappa(b_2 - b_0'') - \kappa^2 b_1 - \kappa' b_0'\right] y^3 \end{aligned} \tag{26}$$

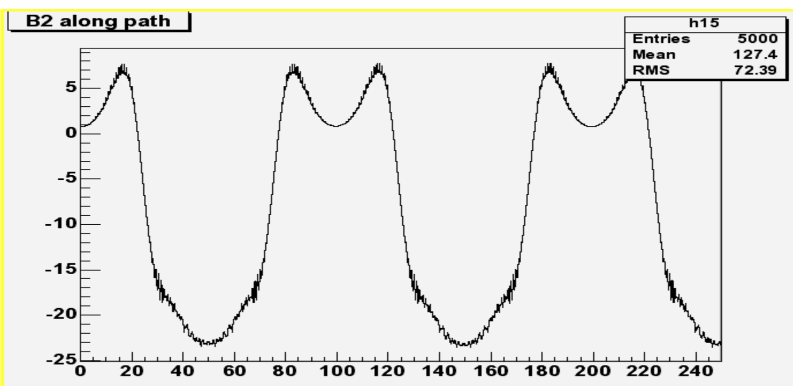
$$\begin{aligned} B_y(x,y,s) &= b_0 + b_1 x - (a_1 + b_s')\,y \\ &+ b_2\,x^2 - \left[2a_2 + \kappa(a_1 - 2b_s') - \kappa' b_s\right] xy \\ &-\frac{1}{2}\left(2b_2 + b_0'' + \kappa b_1\right) y^2 + b_3\,x^3 \\ &-\frac{1}{2}\left[6a_3 + a_1'' + 2\kappa(a_2 + 3\kappa' b_s) - 2\kappa^2(a_1 - 3b_s')\right] x^2 y \\ &-\frac{1}{2}\left[6b_3 + b_1'' + 2\kappa(b_2 - b_0'') - \kappa^2 b_1 - \kappa' b_0'\right] xy^2 \\ &+\frac{1}{6}\left[6a_3 + 2a_1'' + b_s''' + \kappa(4a_2 + 5\kappa' b_s) - \kappa^2(a_1 - 4b_s')\right] y^3 \end{aligned} \tag{27}$$

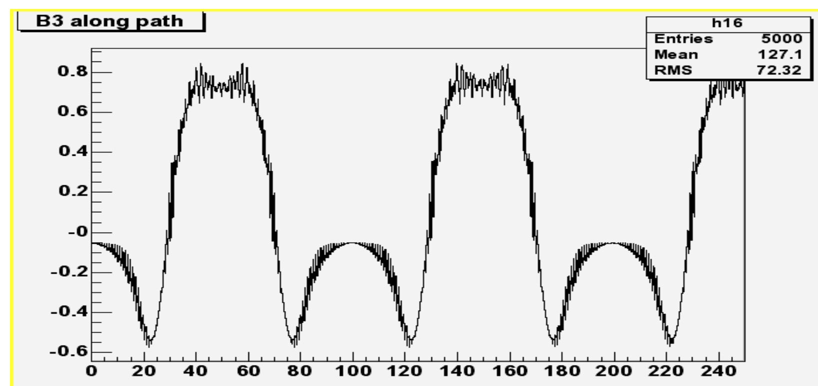
$$\begin{aligned} B_s(x,y,s) &= b_s - \kappa b_s\,x + b_0'\,y \\ &+\frac{1}{2}\left(a_1' + 2\kappa^2 b_s\right) x^2 + \left(b_1' - \kappa b_0'\right) xy - \frac{1}{2}\left(a_1' + b_s''\right) y^2 \\ &+\frac{1}{6}\left(2a_2' - 3\kappa a_1' - 6\kappa^3 b_s\right) x^3 + \left(b_2' - \kappa b_1' + \kappa^2 b_0'\right) x^2 y \end{aligned} \tag{28}$$

b_n along the path









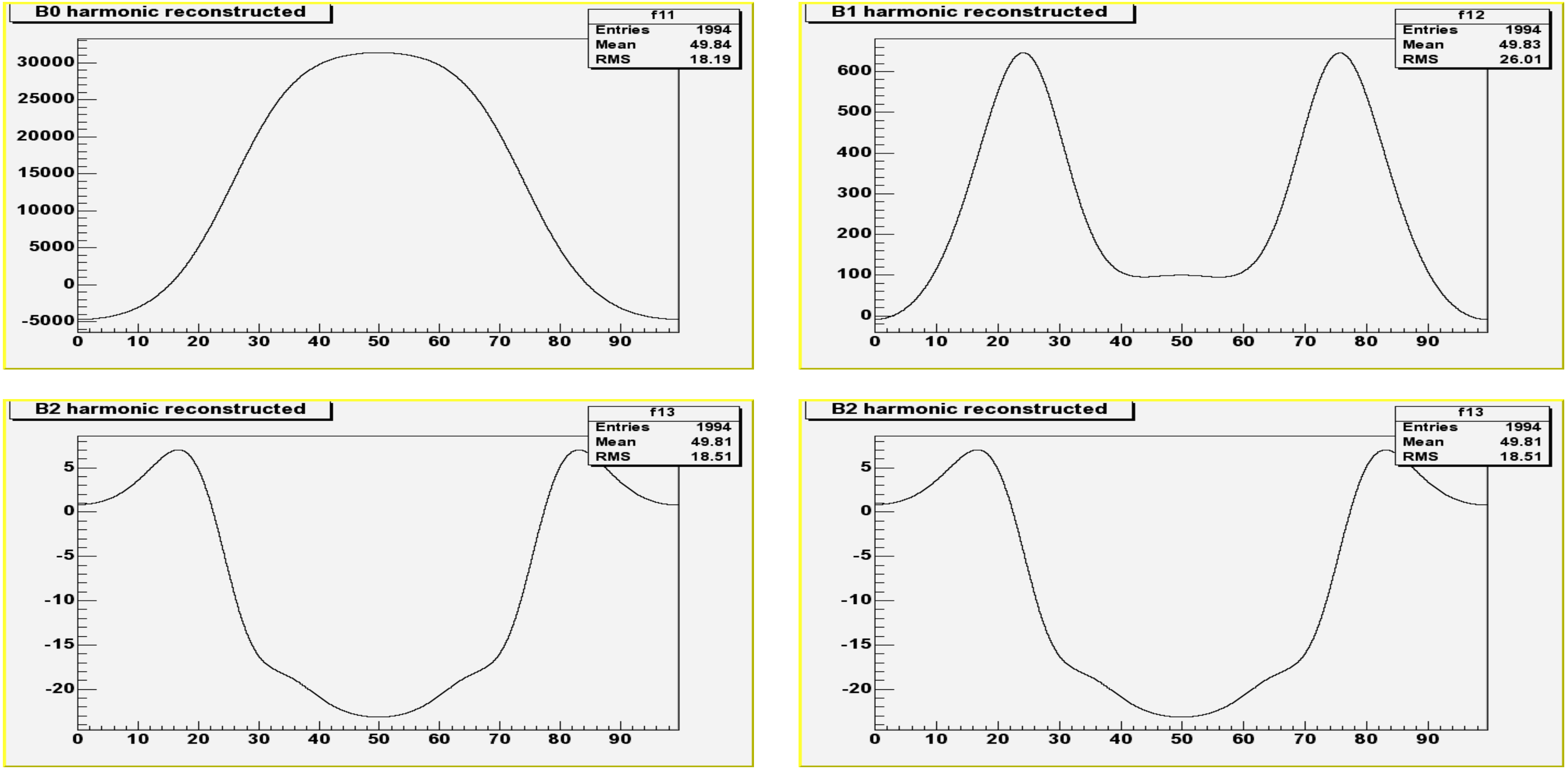
Fourier Expansion of $b_n(s)$

- The $b_n(s)$ can be expanded with a Fourier series:

$$b_n = \Re \sum_{k=0}^{N-1} c_{k,n} e^{-ik\frac{s}{T}} \quad \text{where} \quad c_{k,n} = \frac{1}{T} \int_0^T b_n(s) e^{ik\frac{s}{T}}$$

- These Fourier coefficients can be fed to ICOOL to describe the field with the *BSOL 4* option.
- We use the b_n for $n=0$ to 5.

The b_n Series Reconstructed from the $c_{k,\,n}$ Harmonics as a Verification



Storage Ring Mode

- Modify Harold Kirk’s ICOOL deck to accept the Fourier description of the field.
  - Scale the field to 250 MeV/c on the reference orbit.
    - This is a few percent correction.
- Verify the configuration in storage ring mode.
  - RF gradient set to zero.
  - Material density set to zero.
- Use a sample of tracks with:
  - $\delta x = \pm 1$ mm; $\delta y = \pm 1$ mm; $\delta z = \pm 1$ mm;
  - $\delta p_x = \pm 10$ MeV/c; $\delta p_y = \pm 10$ MeV/c; $\delta p_z = \pm 10$ MeV/c;
  - Also the reference track.

Dynamic Aperture

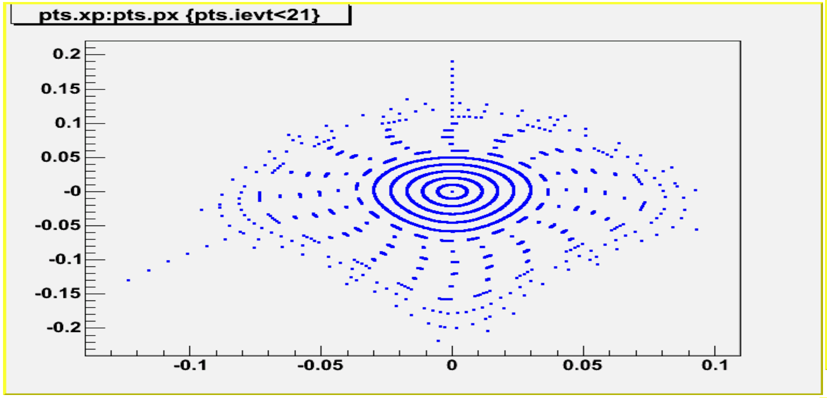
- In order to obtain the dynamic aperture I launched particles at a symmetry point with different start x (y).
- The particle position in x vs p_x (y vs. p_y) was observed as the particle trajectory crossed the symmetry planes.
- I have examined 4 cases:
  - Harold Kirk's original Hardedge configuration.
  - My Hardedge configuration which tries to duplicate Al Garren's lattice
  - My Realistic configuration which tries to duplicate Al Garren's lattice.
  - The Realistic configuration ignoring higher order field components.

Model Parameters

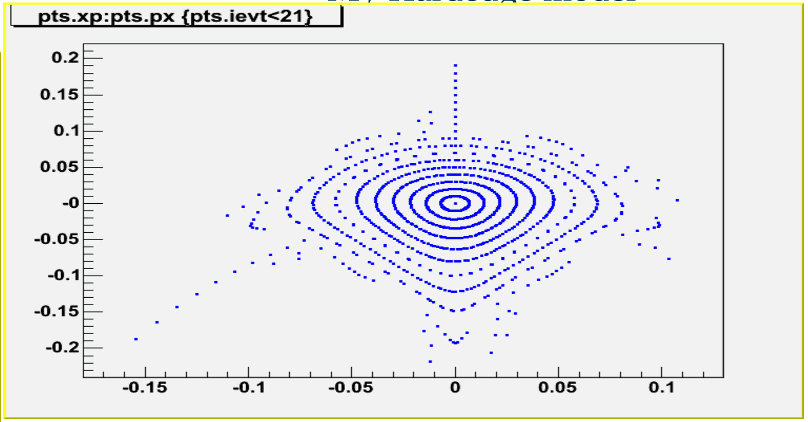
Parameter	Kirk	Kahn
Momentum	0.25 GeV/c	0.25 GeV/c
B_y	2.183 T	2.622 T
Ref. Radius	38.2 cm	31.8 cm
Dipole Length	60 cm	50 cm
Drift Length	27 cm	24.85 cm
Circumference	4.56 m	3.986 m
Edge Matrix Element	1.0844	1.30129
Angle	22.5º	22.48º
Date	July 2003	Nov 2002

Horizontal Dynamic Aperture (x vs. p_x)

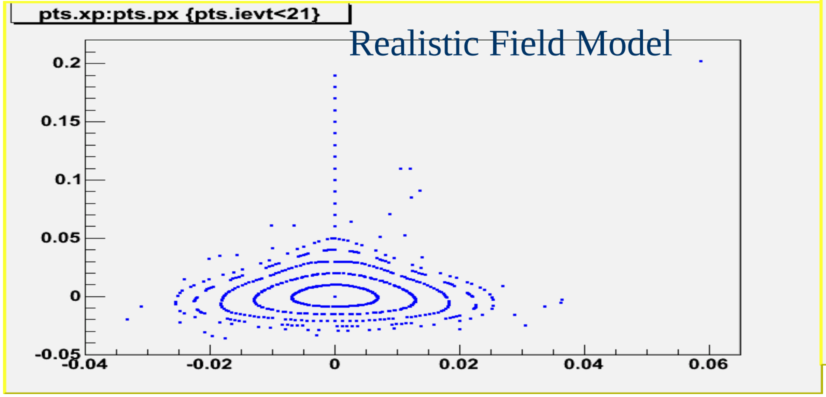
Kirk's Hardedge model

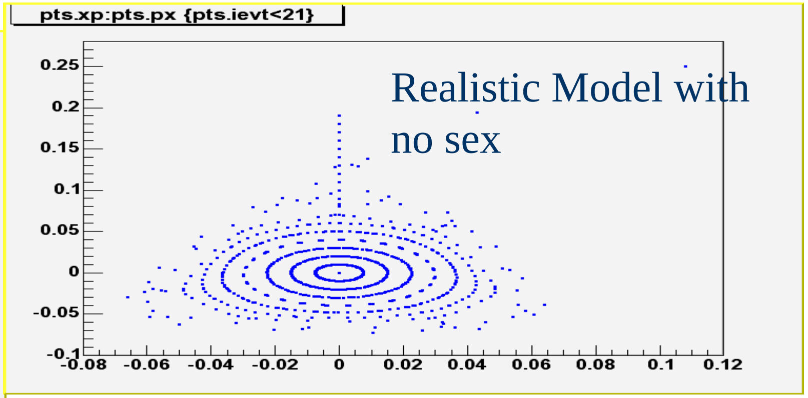


My Hardedge model



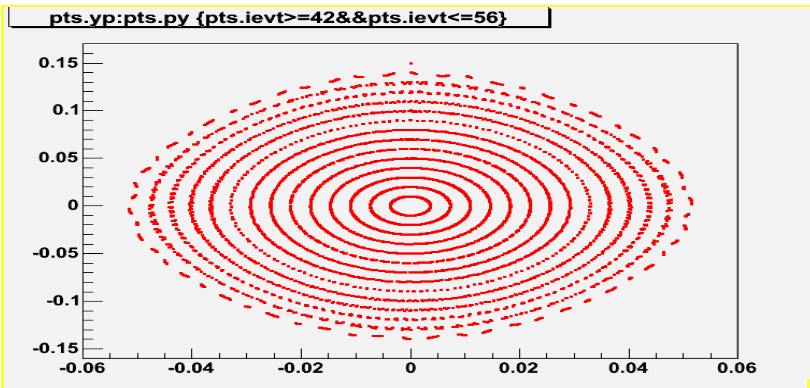
Realistic Field Model



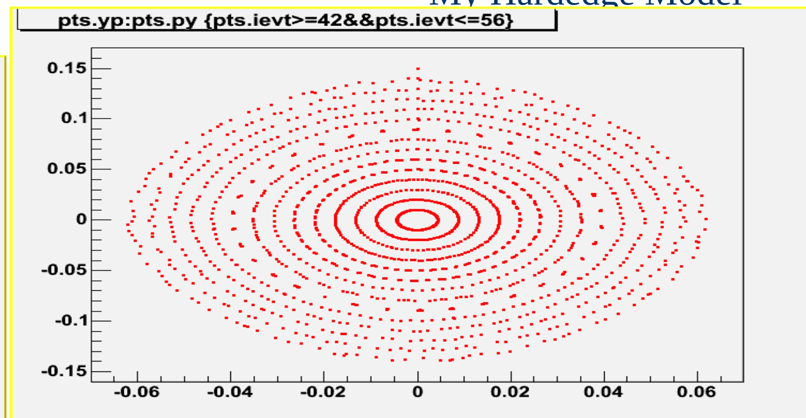


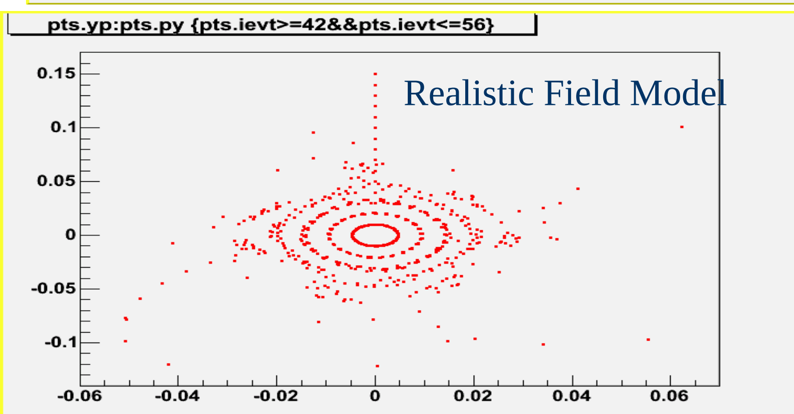
Vertical Dynamic Aperture (y vs. p_y)

Kirk's Hardedge Model

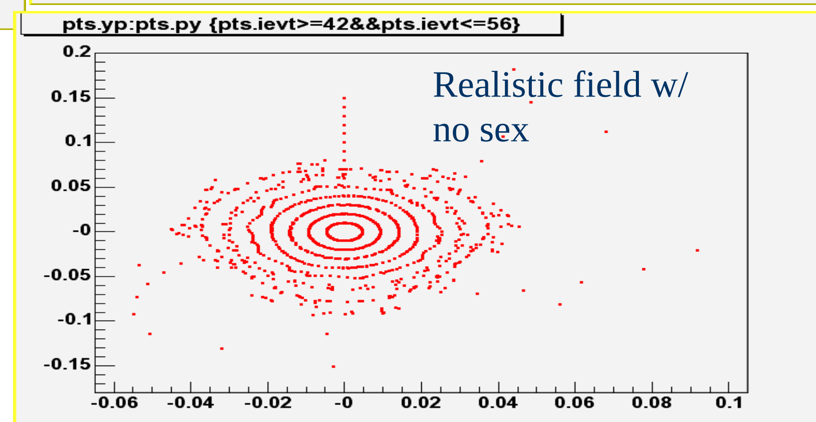


My Hardedge Model





Realistic Field Model



Realistic field w/
no sex

Measure Dynamic Aperture: Counting Rings

	Kirk Hardedge	*Kahn Hardedge*	*Kahn Real*	*No Higher Order*
x P_x	13	9	5	8
y P_y	14	14	4	7

Storage Ring Parameters

- The table below shows the Twiss Parameters as seen in ICOOL for both the *realistic* and *hardedge* models. These were calculated in a manner similar to those shown before
- Both ICOOL models look reasonably comparable to the original SYNCH and TOSCA models.
  - This is extremely encouraging and says that the realistic fields do not significantly alter the lattice!

Parameter	A. Garren Synch	Tosca	Icool Realistic	Icool Hardedge	Icool with No Sex
μ_x	99.8784°	98.38°	105.496°	103.626°	106.313
β_x	37.854 cm	32.3099 cm	34.293 cm	38.8635 cm	33.6023 cm
α_x	0	-0.00124	-0.000461	-0.000576	-0.00593
μ_y	92.628°	100.62°	100.619°	94.9662°	100.865
β_y	56.891 cm	53.9188 cm	54.086 cm	56.9616 cm	53.844 cm
α_y	0	0.0009894	0.000652	-0.000001	0.00597

Conclusion

- We have shown that for the dipole cooling ring that hard edge representation of the field can be replaced by a coil description that satisfies Maxwell's equations.
  - This *realistic* description maintains the characteristics of the ring.
  - This *realistic* description also maintains a substantial fraction of the dynamic aperture.